COMMENTS

On sequences of temporary equilibrium

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C11.1.1. Discussion

The papers by Green and Stigum, appearances notwithstanding, are complementary. They are also important contributions to the theory of temporary equilibrium; a theory of market systems in which the markets for trading commodities may be open each day. This model thus constitutes a generalization of the Arrow–Debreu model. In modeling such an economy, several issues must be faced immediately. One is the possibility that, at some date, the equilibrium price in certain markets may be different from what it was in the past. For example, the price paid today for delivery of goods in 1980 may be different from the price paid three days ago for the same delivery contract. Once this possibility is introduced into the model, it is necessary to introduce expectations (usually on prices) about the future possitilities for trades on the (currently) closed markets. Finally, once such expectations are introduced, speculation becomes not only possible but potentially profitable. Thus, individuals may contract to deliver commodities at some future date which they currently do not own (i.e., they sell short), under the expectation that they can buy up the appropriate amount on some day prior to the date delivery is to be made. Such behavior can lead to an inability to deliver if prices are not as expected, thus causing bankruptcy as a result of past behavior. Bankruptcy in turn may operate to insure that no temporary equilibrium exists, thus exposing a basic deficiency in the model.

Green and Stigum approach these problems in different ways. This can be more clearly seen if we initially consider a result of Arrow and Hahn (1, theorem 7, p. 121). They show that, under acceptable assumptions, a compensated temporary equilibrium exists even if bankruptcy

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is allowed. In general, this compensated equilibrium will not be a market equilibrium; however, it is true that there is a redistribution of initial endowments such that the compensated equilibrium will be a market equilibrium after the redistribution. Although Stigum's paper does more, one of his results provides conditions on preferences and expectations such that the compensated equilibrium is a market equilibrium, *without a redistribution of endowments*. Green, on the other hand, accepts that a redistribution is likely to be necessary and provides a set of institutional rules which will accomplish this.

The contribution of each paper is now evident. From Stigum's work it is obvious that the sufficient conditions required to insure that a sequence of temporary equilibria exists without any consumer becoming bankrupt (what he calls a feasible tree structure of temporary equilibria), in economies with only markets for current goods and securities, are so strong that any possibility that reality is encompassed is effectively eliminated. Thus, Stigum's paper contains the motivation for Green's work. That is, if bankruptcy is as likely a possibility as Stigum's paper indicates, methods for dealing with it must be developed. Green, in his potentially seminal paper, provides us with one possibility.

Each paper is important; however, each has certain weaknesses. It is clear that each author ignored these in order to concentrate on what he considered to be the important issues. Thus my reason for discussing these points is not to be critical of the authors but to indicate the desirability of certain future work. It seems to me that the description of the behavior of equilibria in the presence of inactive markets is one of the main contributions of the theory of temporary equilibrium. Stigum's paper requires all commodity futures markets to be inactive: Green's requires none to be inactive. Each paper suffers a little from these extreme positions. Green's assumption implies that if there is no bankruptcy currently and if consumers expect with certainty that today's (relative) futures prices will be tomorrows' current prices, then expectations will be fulfilled and no bankruptcies will occur in the future. Another problem is that if some futures markets did not exist then the concept of the present value of wealth (defined at current prices) which he uses as a standard of bankruptcy is not defined. Personally I do not think this detracts from his major insights; however, it would be nice to see what happens in a world without complete futures markets. Stigum's assumption (that no commodity futures markets are active) is restrictive in a different sense. In particular it implys that all debt is paid off in 'dollars' (the unit of account). This immediately makes the price level (as opposed to relative prices) important. For example, normalization of first period prices is legitimate only if $V^k(e_{01})$, the dollar amount of securities maturing at time 0 held by consumer k, equals zero for all k. Otherwise, since commodity endowments are positive, the price level in period 1 could be set high enough such that all debt can be paid. (For additional comments on the role of the price level in a 'monetary' economy see Arrow and Hahn [1, pp. 347–369].) Thus assumption II of Stigum that $V(e_{01}) = 0$ is crucial and effectively eliminates the possibility of a past – precisely the phenomena Green introduces in his model. In Green's paper all debt is owed in commodities and, therefore, he does not have to face this problem. Clearly, more research is needed before the precise relationship between and impact of alternative debt forms (money, securities, or commodities) is understood.

Perhaps the most troubling aspect of Green's analysis is that he is only able to show the existence of an approximate equilibrium. This results from the fact that, because of the institutional arrangements to handle bankruptcy, demand correspondences can be non-convex. It is tempting to ask whether it is possible to revise the institutional rules for redistributing endowments in response to bankruptcies in a way which rescues the convexity of the consumers' demand correspondences. Since several of the disagreements between Green and Stigum must be resolved in such a revision, it is of interest to explore some possibilities.

The main disagreement arises over the definition of bankruptcy. Green defines a consumer to be bankrupt at the prices p if his net present value of wealth (both endowments and contracts) is negative when valued at today's current and future prices. Stigum defines a consumer to be bankrupt if he is unable to find someone who is willing to refinance his currently expiring contractual debts. Let us be a bit more precise. Let ${}^t w = (w_t, w_{t+1}, \ldots)$ be the consumer's current and future endowments at t. Let ${}^{t-1}e = ({}^{t-1}e_t, \ldots)$ be the consumer's current and future contracted commitments at date t. (${}^{t-1}e_{\tau k} > 0$ means he holds, at t, contracts for commodity k to be delivered to him at τ.) Given ${}^t w$, ${}^{t-1}e$ and prices ${}^t p$, the consumer must choose at t a vector of trades (current and futures contracts) ${}^t b$ such that

$$ {}^t p \cdot {}^t b \leq 0, \qquad \text{for all } t. \tag{C11.1} $$

(*Note:* if some markets are inactive then the appropriate entries in ${}^tb$ must equal zero.) Having signed these contracts the consumer consumes, at t, $x_t = w_t + {}^{t-1}e_t + {}^tb_t$ and has remaining contracts of ${}^te$ where ${}^te_\tau = {}^{t-1}e_\tau + {}^tb_\tau$ for $\tau \geq t$. He is constrained to choose (for survival reasons)

$$ {}^tx_t \geq 0 \qquad \text{for all } t. \tag{C11.2}$$

Green's definition of bankruptcy can be seen by rewriting (C11.1) as follows:

$$ {}^tp \cdot {}^tx \leq {}^tp \cdot {}^tw + {}^tp \cdot {}^{t-1}e \tag{C11.3}$$

where ${}^tx$ is his planned current and future consumption. A consumer is then bankrupt if there is no consumption *plan* which satisfies (C11.2) and (C11.3) simultaneously. This occurs if and only if the net present value of wealth, ${}^tp \cdot {}^tw + {}^tp \cdot {}^{t-1}e < 0$. This view is certainly consistent with the approach of Debreu [2], where ${}^0e = 0$ and ${}^1x_\tau = {}^tx_\tau$ for all $\tau \geq 1$. (*Note:* if some markets are inactive, it is not clear what 'expected' prices should be used to evaluate ${}^tw$. For our purposes, however, this is a side issue.)

Stiglitz's definition of bankruptcy can be seen by rewriting (C11.1) as follows:

$$ {}^tp_t \cdot {}^tx_t + {}^t\hat{p} \cdot {}^t\hat{b} \leq {}^tp_t w_t + {}^tp_t \cdot {}^{t-1}e_t \tag{C11.4}$$

where ${}^tp = ({}^tp_t, {}^t\hat{p})$ and ${}^tb = ({}^tb_t, {}^t\hat{b})$. A consumer is then bankrupt if there is no consumption vector ${}^tx_t$ satisfying (C11.2) and a trade ${}^t\hat{b}$ which can be completed *in equilibrium* such that (C11.4) holds. This view is consistent with that of Debreu [3] where ${}^tw < 0$ is possible in a limited way which ensures no bankruptcy of this type. However appealing this view might be as a representation of reality, I find it less compelling as a concept of bankruptcy in a *tatonnement* system since it is not independent of the existence of equilibrium. That is, it states an individual is bankrupt if there is no equilibrium such that he is not bankrupt. This seems particularly circular to me.

Another extreme view would be that a consumer is bankrupt if the value of currently maturing obligations ${}^tp_t \cdot w_t + {}^tp_t \cdot {}^{t-1}e_t < 0$. That is, he is given no opportunity to refinance his debt.

We thus have at least three possible views of bankruptcy from which to choose (Green's is an intermediate case). The only reason the choice must be made is that both Green's and Stigum's institutional rules force a consumer to declare bankruptcy if and only if he is bankrupt according to their criterion. A possible way out of this dilemma is to allow the consumer to choose the extent of his default as a decision variable. That is, we allow default plans just as we allow consumption plans. This would be more consistent with the idea of an informationally decentralized market system. Let me try to indicate how this might work, using Green's notations and concepts.

Instead of relying on Green's institutional default rule, $d(p, r) = \min\{\delta \geq 0 \mid p(w + re_+ + (1 - \delta)e_-) \geq 0\}$, we allow each consumer, i, to choose $d^i \in [0,1]$ given prices p and returns ratios r. Once each consumer has done so, new returns ratios are computed, as in Green, and prices are adjusted. Then new demands and default ratios are computed, etc. The only question is by what criterion does a consumer select a d^i? Clearly if there is no penalty connected with $d^i > 0$, and if preferences are monotonic then he would always maximize utility by choosing $d^i = 1$. That is, he would always desire to default on all commitments. On the other hand, if the penalty is severe (as, for example, in Green where $d^i > 0$ implies ${}^t x_\tau = 0$ for all τ), he might never desire to default unless forced to do so.

The first problem, the consumer always defaults, is inherent in any model where contracts are not strictly enforced. It is basically a problem of public goods (more precisely, bads) and involves an element of social trust. In this sense contracts are like money in that money won't be held (contracts won't be signed) unless there is some faith that it can be exchanged (that they will be carried out). Rather than sort this problem out, it is easier to assume that defaulted contracts carry some disutility (because of, say, social norms) to the defaulter. In particular we can let (as in Green) the utility function of a consumer be $u(x_1, y_1, x_2, y_2, \ldots)$ where x_t is consumption at t and $y_t = d_t{}^t p \cdot {}^{t-1}e_-$ is the dollar value of defaulted contracts at date t.

The second problem, that d might always be chosen to be 0, is not as easily solved. It is a problem because each consumer, given the price ${}^t p$, can always *plan* a trade ${}^t b$ which would refinance all his debt. However, there may be no price such that these plans add up (across consumers) to zero in which case there is no equilibrium. If t is the final decision period, the problem disappears since the consumer must choose d and ${}^t b$ such that $x_t \geq 0$. It is easily shown that if the present value of wealth is negative then d must be chosen to be greater than zero. For decision periods prior to the final period, one must introduce enough assumptions on expectations and utility to insure that the consumer does not expect to be able to refinance all of a large current debt. (Green does this through his assumptions on u and Ψ.)

An obvious objection to this model is that default does not require a declaration of bankruptcy (i.e. one need not even partially pay one's creditors except through the economy wide returns ratios). This is also a feature of Stigum's model in which default is paid for by everyone. Green on the other hand extracts the ultimate penalty even if the consumer defaults on only ε"$_0$ of his contracts. Again a middle road might help re-establish convexity of Green's demand correspondence while forcing the defaulter to bear more of the burden of his actions. One possibility is to require that some percentage of defaulted contracts be covered by the consumer's own assets. Remembering that d is the percentage defaulted on, let $t(d)$ be the institutionally predetermined percentage of assets required to cover default. Then, a consumer's budget constraint would be: $px \leq (1 - d)pe_- + (1 - t(d))(pre_+ + pw)$. In order for a consumer to be able to always attain a non-negative wealth position we would need $t(d) < d$. If $t(d)$ is continuous and convex in d then demand correspondences should be well behaved. That is, under assumptions similar to Green's, demands and default ratios should be upper-semi continuous, convex, non-empty correspondences of prices and returns ratios. If the formula for computing the returns ratio is then suitably adjusted, one should be able to establish the existence of temporary equilibrium. This remains to be shown.

I have concentrated on the aspects of each paper dealing with the question of existence. However, once existence is established, it is interesting to inquire about its optimality properties. Stigum's paper is one of the few to do this for economies with sequences of temporary equilibria. His results (embodied in theorems 11.3 and 11.4 and some counterexamples) deserve emphasizing. Briefly, he shows that, in general, a particular sequence of temporary equilibria is not Pareto-optimal *ex post*.

However, he does provide, without proof, very restrictive conditions such that *ex post* Pareto optimality does obtain. He also demonstrates that there exists a set of expectations and a redistribution of first-period purchasing power such that any *ex post* Pareto-optimal allocation can be supported by a sequence of temporary equilibria. That is, optima are equilibria (given a redistribution of endowments and specific expectations) but equilibria may not be optima except in fortuitous circumstances. Finally, he indicates (in theorem 11.4) that each temporary equilibrium plan is *ex ante* Pareto-optimal given the expectations of consumers. Thus, he has illuminated the complex relationship between sequences of temporary equilibria, *ex ante* Pareto-optimal plans, *ex post* Pareto-optimal allocations, and price expectations.

In summary, I consider both of these papers to be excellent contributions to our knowledge about the performance of market economies in which temporal sequences of markets exist. Stigum's work establishes that in general, 'a price mechanism confined mainly to current markets for current goods is likely to go astray'. (This is a view attributed to Keynes by Arrow and Hahn [1, p. 347].) Green's work initiates the important task of revising the usual rules of market behavior to allow sequences of temporary equilibria to proceed in an orderly fashion. Clearly this work has just begun.

References

[1] Arrow, K. J. and Hahn, F. H. *General Competitive Analysis*. Holden Day: San Francisco (1971).

[2] Debreu, G. *Theory of Value*. John Wiley and Sons, Inc.: New York (1959).

[3] Debreu, G. New concepts and techniques for equilibrium analysis. *International Economic Review*, 3 (September 1962), 257–273.